PROPAGATION OF COMBUSTION WAVES IN BUBBLY MEDIA

G. S. Sukhov and L. P. Yarin

UDC 662.612+536.46

The propagation of combustion waves in bubbly media is of appreciable interest for numerous technical applications. The possibility of the existence of such waves was first indicated in [1-3], where investigators substantiated the physical mechanism of the phenomenon and used asymptotic methods of combustion theory to study laws governing the self-sustaining wave process with a chemical reaction propagating in an active fluid containing bubbles of a gaseous oxidant. Subsequent studies involved generalizations to the case of the propagation of polymerization waves in a liquid with liquid monomer occlusions distributed within it [4, 5].

Zhizhin [6] qualitatively analyzed the system of equations proposed in [1] to describe the combustion of bubbly media. A determination was also made of the rate of propagation of the chemical reaction in a medium containing particles of an active disperse phase. However, [6] contains several unsubstantiated statements concerning not only the solution of the particular problem, but also the overall approach taken to the analysis of combustion-wave propagation in reacting media.

Of foremost concern here is proof of the existence of a steady-state wave solution with a chemical source. As was shown in [7], searching for such a solution is justified only if the existence of the chemical source can be ignored on an infinite interval of space ahead of the wave $-\infty \leq x \leq x_0$† sufficiently far from the region of the active reaction. Physically, this means that the initial formulation of the problem must include the well-known condition which "cuts off" the kinetic function in the neighborhood of the initial state. It follows directly from this that the initial system of differential equations must be broken down into two subsystems in which $W = 0$ (the chemical kinetics "cutoff" zone) and $W > 0$ (the reaction zone). The solution in the region $W = 0$ should correspond to the section of the integral curve lying in the plane $c = c_0$, while in the region $W > 0$ the solution corresponds to the section of the curve lying in the phase $0 < c < c_0$. Proof of the existence of a stationary wave structure should reduce to proof of the existence of at least one value of combustion propagation rate $u > 0$ at which the above-indicated sections of the integral curves join together at the "cutoff" point of the kinetic function ($\theta_1 = \theta_{1*}$, $\theta_2 = \theta_{2*}$, $c = c_0$) [7]. Although Zhizhin [6] proposed the "cutoff" principle, it was not realized in the qualitative analysis of the differential equations. Evidence of this comes from the fact that there is no section of the integral curve in the plane $c = c_0$ which corresponds to the "cutoff" zone (Fig. 1 in [6]). This in turn casts doubt on the final results of the study. Also unconvincing is the assertion of the existence of a continuous integral transition from the initial state to the final state, this conclusion having been reached on the basis of analysis of only small neighborhoods of the singular points.

One of the central premises of [6], concerning the decisive role of asymptotic decay of the reaction in the mechanism of propagation of combustion waves, is untenable. By no means would ignoring the weak reaction in the zone where combustion proceeds to completion lead to "rejection of the sought wave solution" (to use the author's phrase). It would mean only that the smooth integral curve is approximated in the neighborhood of the final singular point $\theta_1 = \theta_2 = \theta_m$, $c = 0$ as a piecewise-continuous curve with sections lying within the phase space $0 < c < c_0$ and in the plane $c = 0$ as occurs in the neighborhood of the initial singular point.

We take exception with the calculation of the structure of the wave in the zone associated with asymptotic damping of the reaction. Only the concentration of the reactant changes

---

†Here and below, we use the notation from [6].

St. Petersburg. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 2, pp. 165-166, March-April, 1992. Original article submitted January 9, 1991.

slightly in this zone; the changes in the temperatures of the phases may be substantial within the zone. Thus, it is incorrect to attempt to find a solution by linearizing the initial system of differential equations in the neighborhood of the final state. This approach can lead to errors which cannot be predicted. One consequence of this in particular might be inadequancy of the regions of existence of the solution in [1, 6]. This could in fact explain the differences seen in calculations of the rate of propagation of combustion in the hexane—oxygen system. The reliability of the analysis in [1] was proven in [3] by wave structure calculations performed on a computer.

In conclusion, we note that the assertion made in [6] regarding the error of the analysis of wave stability supposedly performed in [1, 2] is also untenable. In fact, the problem of the stability of nucleate combustion was never rigorously addressed in [1, 2]. The authors of the latter studies qualitatively analyzed only steady-state solutions, and they reserved the term "unstable" for one of the solutions that did not make sense physically. In later publications on the same subject (such as [8]), this term was generally avoided.

There was also no error in the heat-conduction equation for the liquid in [2]. In accordance with the notation adopted in [2], in the conductive term of this equation we introduced the effective thermal conductivity of the liquid phase λ. This quantity is related to the physical value of this parameter λ^0 by the obvious equality $\lambda = (1 - \varphi)\lambda^0$, where φ is the volume content of the gas phase.

LITERATURE CITED

1. G. S. Sukhov and L. P. Yarin, "Combustion waves in bubbly media," Dokl. Akad. Nauk SSSR, 256, No. 2 (1981).
2. G. S. Sukhov and L. P. Yarin, "Laws of combustion of bubbly media," Fiz. Goreniya Vzryva, No. 3 (1981).
3. V. N. Likhachev, G. S. Sukhov, and L. P. Yarin, "Certain laws governing combustion waves in bubbly media," in: Chemical Physics of Combustion and Explosion Processes. Combustion of Condensed Systems [in Russian], Chernogolovka (1986).
4. P. L. Gusika, "Theory of preparation of a polymerization front in a suspension," Khim. Fiz., No. 2 (1982).
5. P. L. Gusika, "Existence and uniqueness of the steady-state solution of a problem on the propagation of an exothermic wave in a two-phase medium," Fiz. Goreniya Vzryva, No. 6 (1982).
6. G. V. Zhizhin, "Self-similar preparation of chemical reactions in disperse media," Prikl. Mekh. Tekh. Fiz., No. 6 (1988).
7. Ya. B. Zel'dovich, G. T. Barenblatt, V. B. Librovich, and G. M. Makhviladze, Mathematical Theory of Combustion and Explosion, Nauka, Moscow (1980).
8. L. P. Yarin and G. S. Sukhov, Principles of a Theory of Combustion in Two-Phase Media [in Russian], Énergoatomizdat, Leningrad (1987).

ERRATUM

In the article "Shock Wave Front from an Underground Explosion," published in Vol. 32, No. 6, November-December, 1991, on p. 843, the first part of Eq. (2.1) should read:

$$\int_{r_3}^{R} \rho r^2\,dr = \frac{\rho_0\left(R^3 - a_3^3\right)}{3k}, \tag{2.1}$$

On p. 846, the y axis of Fig. 6 should read p_2, GPa.